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EDITORIAL
A SPECIAL ISSUE ON RECENT ADVANCES IN VECTOR AND SET OPTIMIZATION
IN HONOR OF THE 70TH BIRTHDAY OF PROFESSOR CHRISTIANE TAMMER

The study of vector and set optimization is now under the spotlight of optimization research due to it wide application in various fields, such as economics, finance, image processing and so on. Set optimization is concerned with the problems that include set functions in the objective and constraints. The main approaches for defining solution concepts for set optimization problems are the vector approach, the set approach, and the lattice approach. The objective of this special issue is to bring together original research on vector and set optimization, highlighting new developments in vector and set optimization and fostering cross-disciplinary exchanges.

This special issue starts with the paper "ℓ_0-Norm Multiobjective Optimization Models Motivated by Applications to Proton Therapy", by X. Cong, X. Ding, B. Mordukhovich, A.V. Nguyen, and L. Zhao. They investigated single-objective and multiobjective optimization problems involving the ℓ_0-norm function, which is nonconvex and nondifferentiable. They proposed several algorithms of the subgradient type and conduct their convergence analysis. They also presented numerical examples, which reveal some characteristic features of the proposed algorithms and their interactions with the gradient descent.

In the paper "Approximate KKT Conditions for Generalized Bilevel Optimization with a Variational-Inequality Lower Level", authored by R. El Idrissi, E.M. Kalmoun, L. Lafhim, necessary optimality conditions for nonsmooth generalized bilevel optimization problems in which the lower level is a variational inequality were investigated. They derived Karush–Kuhn–Tucker (KKT) type conditions via the basic (limiting) subdifferential. They introduced an approximate KKT (AKKT) framework tailored to this nonsmooth, hierarchical setting and proved that every KKT point is an AKKT point, while the converse may fail in general, as demonstrated by a counterexample. Their results positioned AKKT conditions as a robust tool for bilevel analysis and computation when smoothness or classical constraint qualifications are unavailable.

S. Bai, A.A. Khan, C. Tammer, and A. Zălinescu, in "A Unified Stochastic Approximation Framework for Mean-Square and Almost Sure Convergence with Rates in Multivalued Variational Inequalities", presented a unified stochastic approximation framework for solving multivalued variational inequalities in real separable Hilbert spaces. They proposed an iterative

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projection scheme that operates over a sequence of expanding closed convex subsets, a formulation motivated by variable-mesh discretization strategies in infinite-dimensional optimization. They established the strong convergence of the generated sequence, in both the mean-square and almost sure senses, to the limit of the solutions of the discretized problems, which is a solution of the original variational inequality under standard approximation conditions. They also derived explicit rates of convergence for both modes of convergence, and investigated the ill-posed case of vanishing regularization.

The contribution, "Unification of Semicontinuities and Applications in Analyzing Stability for Set Optimization Problems", by L.Q. Anh, L.V. Day, L.T.V. Khanh, and X. Zhao, mainly investigated the upper and lower semicontinuity of a set-valued map and discussed properties and characterizations. The authors proposed a unification of the set less relations via the Minkowski difference and examine its properties. They introduced new approaches and analyzed the convergence behavior of interval optimal control problems and the irrigation management model under uncertainties in rainfall and cost.

The authors, M.A.T. Ansary and Joydeep Dutta, in this paper, "Newton's Methods Using Bregman Functions for Convex Multiobjective Optimization", studied a multiobjective convex optimization problem constrained by a closed convex set. They developed a Newton type method for such a problem. They proved that, under mild assumptions, their iterative sequence converges a critical point, a weak Pareto minimizer.

In the paper, "On DC Mappings between Normed Spaces and Applications in DC Vector Optimization", L. Huerga, B. Jiménez, and V. Novo introduced several variants of the most known notions of vector DC mappings defined on normed spaces. Their main results are generalizations to the vector case of the well-known characterizations of approximate and exact global minima of an unconstrained scalar DC optimization problem due to Hiriart-Urruty [J.B. Hiriart-Urruty, Generalized differentiability, duality and optimization for problems dealing with differences of convex functions, In: J. Ponstein, (eds) Convexity and Duality in Optimization, Lecture Notes in Econom. and Math. Systems, vol. 256, pp. 37-70, Springer, Berlin, Heidelberg, 1985].

J. Li, A. Petrusel, C.F. Wen, and J.C. Yao, the authors of "Covering Constants for Linear and Continuous Mappings with Applications to Stochastic Systems of Linear Equations", introduced the concept of locally or globally stochastic systems of linear equations in $\mathbb{R}^n$. They employed the Mordukhovich derivatives to precisely find the covering constants for some linear and continuous mappings in $\mathbb{R}^n$. They proved the existence of solutions to some locally or globally stochastic systems of linear equations in $\mathbb{R}^n$. In addition, they also provided some examples for finding solutions of some stochastic systems of linear equations in $\mathbb{R}^n$.

The paper "Convex Reformulation of a 1R1C Smart Thermostat Thermal Model", by J. Arias, A.A. Khan, and M. Sama, established a reformulation in terms of three parameters that retains physical interpretability, and proved existence, stability, and differentiability results. In particular, an explicit characterization of the Hessian of the reformulated objective map was obtained. The authors proveed strict convexity over a range of regularization parameters and demonstrted how, under a compatibility condition between both models, their leads to uniqueness, or improved approximation when combined with a priori information, for the general four-parameter model.

In his contribution "Pessimistic and optimistic robust solutions with preference structures", César Gutiérrez introduced pessimistic and optimistic robust solution concepts for uncertain vector and set optimization problems with preference structures, providing new insights into minmax and minmin solutions. The paper develops a scalarization scheme for problems with variable domination structures and characterizes robust solutions for uncertain set optimization problems with finitely many scenarios.

All the contributions in this special issue are due to experienced researchers and many of them have shared with Professor Christiane Tammer an intense scientific and personal collaboration. We want to thank all the Authors and Reviewers that made this Special Issue possible. It has been an honor for us to act as Guest Editors of this Special Issue and contribute to the 70th birthday of Professor Christiane Tammer. We hope that the readers will enjoy it.

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