

**EDITORIAL**  
**A SPECIAL ISSUE ON RECENT TRENDS IN NUMERICAL ALGORITHMS AND THEIR APPLICATIONS**

In the modern information age, numerical algorithms receive much attention than ever before due to their wide applications in applied mathematics, management science, computer science, medicine, economics, and other engineering fields. This special issue focuses on the recent trends in various numerical algorithms and their applications. The contributions to this special issue include optimization methods, models, and algorithms that deal with various pure and applied problems emerged in the last decades.

This special issue is comprised of nine articles whose contributions are as follows:

K. Guo and C. Zhu, in the paper “On the linear convergence of a Bregman proximal point algorithm,” studied a Bregman proximal point algorithm for a convex optimization problem. They analyzed the linear convergence rate and establish the linear convergence of the iterative sequence of the algorithm.

In the paper “Multiple-sets split quasi-convex feasibility problems: Adaptive subgradient methods with convergence guarantee,” Y. Hu, G. Li, M. Li, and C.K.W. Yu considered a multiple-sets split quasi-convex feasibility problem (MSSQFP), which is to find a point such that itself and its image under a linear transformation fall within two families of sublevel sets of quasi-convex functions in the space and the image space, respectively. A unified framework of the adaptive subgradient methods with general control schemes was proposed to solve the MSSQFP. They established the quantitative convergence theory of adaptive subgradient methods with several general control schemes. An interesting finding was disclosed by the iteration complexity results that the stochastic control enjoys both advantages of low computational cost requirement and low iteration complexity.

The paper “Multi-objective optimization of a nonlinear batch time-delay system with minimum system sensitivity” by L. Wang *et al.* focuses a nonlinear time-delay dynamic (NTDD) system with uncertain time-delay in batch culture of glycerol bioconversion to the 1,3-propanediol (1,3-PD) induced by *Klebsiella pneumoniae*. They designed an optimization scheme for the NTDD system with the aim of balancing two competing objectives: (i) system cost (the relative error between experimental data and the output of the mathematical model); (ii) system sensitivity (the variation of the system cost with respect to uncertain time-delay). A multi-objective optimization problem (MOOP) governed by the NTDD system was converted into a sequence of single-objective optimization problems (SOOCs) by using convex

weighted sum and modified normal boundary intersection methods. By incorporating the time scaling transformation, the constraint transcription and locally smoothing approximation, a parallel hybrid SOOCP solver was developed based on gradient-based method and genetic algorithm.

G.M. Lee, G.S. Kim, and M.H. Kim, in the paper “Linear Fractional Optimization Problems on Jordan Euclidean Algebras,” considered a linear fractional optimization problem (LFOP) defined on an Euclidean Jordan algebras. They obtained an optimality theorem for the LFOP, which holds without any constraint qualification. Moreover, they formulated the non-fractional dual problem of the LFOP and then proved the duality theorems (weak duality theorem and strong duality theorem), which hold without any constraint qualification. They also characterized the solution set of the LFOP by using the optimality conditions.

In the paper “Constraint qualifications in nonsmooth optimization: Classification and inter-relations,” Rimpi and Lalitha systematically studied various constraint qualifications for a nonsmooth optimization problem constrained by inequality constraints where the functions involved are locally Lipschitz continuous. They classified the constraint qualifications into four levels by using the inclusion relations among the cones of interior constrained directions, feasible directions, attainable directions, tangent directions, and locally constrained directions. Numerous inter-relationships between the constraint qualifications are summarized schematically. They further discussed the nature of various cones of the feasible set by assuming the constraint functions to be semilocally convex, and established the equivalence among some of the constraint qualifications.

The goal of the work “Approximate elements for set optimization problems with respect to variable domination structures” by E. Köbis and M.A. Köbis is to consider the set optimization problems governed by the upper set less relation introduced by Kuroiwa. They extended some known results from the literature to a setting that includes (a) variable domination structures and (b) approximate solution concepts, and showed characterization results by means of inequalities involving duality products. The used notion of optimality is the set approach, and the variable domination structure builds upon comparisons solely dependent on elements in the objective space but include the two-argument case.

J. Teng, K. Guo, and L. Wang, in the article “Dynamic optimization with a non-smooth LPV system in aero-engine transition state acceleration process,” studied the dynamic optimization of the acceleration process in the engine transition state. They considered a non-smooth linear parameter variation (LPV) system with nonlinear terms as a mathematical model for this acceleration process. The effectiveness and superiority of their proposed algorithm is computationally verified by using the LPV model identified from the actual data.

X. Xue, C. Liao, M. Li, and C. Lu, in their work “Coderivatives and Aubin properties of solution mappings for parametric vector variational inequality problems” investigated the sensitivity analysis for a parametric vector variational inequality problem in finite-dimensional spaces by using advanced tools in modern variational analysis and generalized differentiation. They mainly focused on computing the coderivatives of the solution mapping in the parametric vector variational inequality problem and then applied them to the verifiable conditions for the Aubin property of the solution mapping.

The primary focus of the paper, “A homotopy method for multikernel-based approximation” by Y. Lin, Y. Wei, and Q. Ye, is to propose several numerical tricks and show some related propositions and theorems to deal with the considerable computation resources required to solve the multilinear system associated with the multikernel-based approximation method. Moreover, a homotopy method is proved

to be suitable for the multikernel-based approximation method, including the global convergence and the locally superlinear convergence. They also discussed some problems behind the choice of the truncated length  $M$  and the low-rank approximation.

In conclusion, we express our most sincere gratitude to all the authors and the referees who contributed to this special issue, and we hope that the readers will enjoy it.

Sun Young Cho

Research Center for Interneural Computing  
China Medical University Hospital, China Medical University, Taichung, Taiwan  
E-mail address: ooly61@hotmail.com, sy.cho@mail.cmuh.org.tw

Ke Guo

School of Mathematics and Information  
China West Normal University, Nanchong, China  
E-mail address: keguo2014@126.com

Jen-Chih Yao

Research Center for Interneural Computing  
China Medical University Hospital, China Medical University, Taichung, Taiwan  
E-mail address: yaojc@mail.cmu.edu.tw