

**EDITORIAL**  
**A SPECIAL ISSUE ON THE THEORY AND NUMERICAL METHODS FOR  
VECTOR OPTIMIZATION PROBLEMS WITH RESPECT TO VARIABLE  
DOMINATION STRUCTURES**

The objective of this special issue is to present advances in the applications of variable domination structures in vector optimization from a theoretical and numerical perspective. Vector and set optimization with variable domination structures is a growing and expanding field of applied mathematics with applications in medicine, imaging science, psychology, behavioural sciences, logistics, and uncertain programming. Hereby, one deals with optimization problems where the domination structure is given by a set-valued map acting between abstract or finite dimensional spaces. The concept of variable domination structures can be viewed as a generalization of the solution concept with fixed domination structures in multi-objective decision-making problems. It is our great pleasure to discuss the contributions of the finally selected papers below.

This special issue starts with the contribution “A Steepest Descent-like Method for Vector Optimization Problems with Variable Domination Structure” by G. Bouza and Chr. Tammer. This paper proposes a steepest descent-like method for computing nondominated solutions of smooth unconstrained vector optimization problems with variable domination structure. The authors demonstrate that every accumulation point of the generated sequence satisfies a first order necessary condition. The consequences of this fact are discussed in the convex case.

In the paper “Approximate Efficiency in Set-Valued Optimization with Variable Order” by M. Durea, E.-A. Florea, D.-E. Maxim, and R. Strugariu, the authors study constrained set-valued optimization problems with variable order. The aim of this contribution is to deduce conditions for stability of minima of such problems at the perturbations of the objective map and the set of constraints, and furthermore, to study certain possibilities of recovering optimization problems with fixed order. The results are employed in order to derive optimality conditions for constrained set-valued optimization problems with variable order. Four types of cone enlargements are the main tools for deriving the results.

The work “Optimal Payoffs for Directionally Closed Acceptance Sets”, authored by M. Marohn and Chr. Tammer, considers directionally closed acceptance sets in the linear space of capital positions. Assuming finitely many eligible assets, the decision maker of a financial institution has to decide how to invest into these assets to secure acceptability for the financial position, meaning that the resulting capital position belongs to the acceptance set. The authors

study the risk measure that describes the costs for reaching acceptability in the linear space of capital positions. As one conclusion, the authors motivate the use of a variable domination structure in the definition of the acceptance set, that then may vary in dependence of the origin capital position of the financial institution.

In the contribution entitled “Perturbation Analysis of Global Error Bounds for Piecewise Linear Conic Inequalities”, G. Yang and X.Y. Zheng discuss piecewise linear systems of conic inequalities. In particular, the stability of its global error bound is established when the piecewise linear vector-valued objective function undergoes small linear perturbations. This is a useful and practicable extension from the linear case to the piecewise linear one.

Jauny, D. Ghosh, A. Upadhyay, and Q.H. Ansari introduce a trust-region interior-point technique to generate the Pareto optimal solution for multi-objective optimization problems in their work “A Trust-region Interior-point Technique to Solve Multi-objective Optimization Problems and its Application to a Tuberculosis Optimal Control Problem”. The authors utilize the well-known Pascoletti-Serafini scalarization technique to convert a multi-objective optimization problem into a set of single-objective optimization subproblems. Then, the subproblems are solved by a trust-region interior-point method. As an application, they apply the proposed algorithm to solve an optimal control problem for a tuberculosis model.

In the paper “A Nonmonotone Gradient Method for Constrained Multiobjective Optimization Problems”, X. Zhao, J.-C. Yao, and Y. Yao consider a nonmonotone gradient method for smooth constrained multiobjective optimization problems. Under mild assumptions, the authors show the Pareto stationarity of the accumulation point of the sequence generated by this method, while the convergence of the full sequence to a weak Pareto optimal solution of the problem is proven when the function is convex. The linear convergence of the function value sequence to the optimal value is provided under some assumptions on the gradients of the objective functions and the search directions.

J. Liu, J. Zheng, X. Zhang, Z. Wan, and J. Chen propose a new accelerated positive-indefinite proximal linearized alternating direction method of multipliers in their contribution “A Class of Accelerated Negative-definite Proximal ADMM for Constrained Separable Convex Optimization Problems”. The Alternating Direction Method of Multipliers (ADMM) is successfully used in various fields, such as statistical learning, computer vision, image processing, wireless network, and distributed network. The authors make use of the techniques of extrapolation and increasing penalty parameter. The nonergodic convergence rate  $O(1/k)$  in the sense of objective values and the nonergodic convergence rate  $O(1/k)$  in the sense of iterative sequence of the proposed method as well as the upper bound of the violation of constraints is obtained. Numerical experiments illustrate the efficiency of the proposed method.

The special issue ends with “A First Bibliography on Set and Vector Optimization Problems with Respect to Variable Domination Structures” by E. Köbis, M. A. Köbis, and Chr. Tammer. This bibliographical paper lists in alphabetical order 102 papers dealing with variable domination structures and its applications and numerics. A classification is provided in terms of (set-valued) vector (quasi-) equilibrium problems with variable domination structures, inclusion problems with moving cones, vector / multi-objective optimization problems with variable domination structures, vector complementarity problems with variable domination structures, vector variational inequalities, applications and algorithms.

We, as the guest editors, would like to thank all the authors who made a contribution to this special issue and all the reviewers who kindly accepted the invitation to provide their expertise and gave constructive comments.

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