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EDITORIAL
A SPECIAL ISSUE ON VECTOR AND SET OPTIMIZATION IN APPLICATIONS

Vector and set optimization represent modern research directions in applied mathematics with important applications, especially in economics, financial mathematics, risk theory, and medicine. The objective of this special issue is to present modern research results in the fields of vector and set optimization including their applications. It is a great pleasure for us to briefly introduce the contributions for our special issue.

This special issue starts with the contribution ”Vectorial penalisation in vector optimisation in real linear-topological spaces“ by C. Günther, E. Köbis, P. Schmölling, and C. Tammer. This paper presents a vectorial penalization approach for vector optimization problems in which the vector-valued objective function acts between real topological-linear spaces, where the image space is partially ordered by a pointed convex cone. The penalization approach replaces the original constrained vector optimization problem (with not necessarily convex feasible set) by two unconstrained vector optimization problems, where in one of the two problems a penalization term with respect to the original feasible set is added to the vector objective function. For deriving the main results, a generalized convexity (quasiconvexity) notion for vector functions is supposed.

The paper ”Further results on quasi efficient solutions in multiobjective optimization“ by César Gutiérrez Vaquero, is dealing with quasi efficient solutions of a vector optimization problem. The objective function is acting between finite dimensional spaces and the ordering in the image space is defined by a pointed convex cone. Several new concepts of quasi efficient solutions are introduced and their basic properties are derived in this setting. Linear scalarization results are discussed that characterize the introduced quasi efficient solutions by solutions of scalar optimization problems for convex problems.

The contribution ”Convergence rates for nonlinear inverse problems of parameter identification using Bregman distances“ by D. N. Hao, A. A. Khan, and S. Reich is devoted to the establishment of new convergence rates for the nonlinear inverse problem concerning the identification of variable parameters in an abstract variational problem. The authors employ the energy least squares and output least squares methods to discuss the inverse problem in an optimization framework. In terms of the re-nowned Bregman distance associated with a convex regularizer, the convergence rates are given.

The paper ”On set-valued discrete dynamical systems“ by E. Hernandez and J. Peran deals with set-valued discrete dynamical systems with the aim of establishing a general framework and unifying some results and definitions in the literature. The authors generalize dynamical results in terms of single valued maps by the weaker assumptions on continuity.

In the contribution, entitled "Existence results and optimization over the set of efficient solutions in vector-valued approximation theory" by M. Isyaku, C. Tammer, and A. Farajzadeh, existence results for vector-valued optimization problems and corresponding results for associated scalarized problems are derived. The results are applied to special vector-valued approximation problems, especially to multiobjective location problems where the whole set of efficient solutions can be generated by a geometric primal-dual algorithm. Moreover, the nonlinear scalarizing functional introduced by Gerstewitz is used to perform an optimization on the generated set of efficient solutions according to the preferences of a decision maker.

In the paper "Optimality conditions in optimization under uncertainty" by E. Köbis and C. Tammer, scalar optimization problems under uncertainty with infinite scenario sets are considered. Methods from vector optimization in general spaces, set-valued optimization and scalarization techniques are applied to derive necessary optimality conditions for solutions of robust counterpart problems.

In the paper "Newton's method for uncertain multiobjective optimization problems under finite uncertainty sets" by S. Kumar, M. A. T. Ansary, N. K. Mahato, D. Ghosh, and Y. Shehu, the authors develop Newton's method for generating solutions of the robust counterpart of a multiobjective optimization problem under uncertainty. The uncertainty set is supposed to be nonempty and finite. The robust counterpart of the multiobjective optimization problem under uncertainty is considered as the minimum of objective wise worst case, which is a nonsmooth deterministic multiobjective optimization problem. The convergence of the Newton's algorithm for the robust counterpart is justified under some usual assumptions.

G. Kumar and J.C. Yao discuss in their paper "Fréchet subdifferential calculus for interval-valued functions and its applications in nonsmooth interval optimization" the notion of Fréchet subdifferentiability or gH-Fréchet subdifferentiability. The authors explore its relationship with gH-differentiability and derive various calculus rules for gH-Fréchet subgradients of extended interval-valued functions. Employing the proposed notion of subdifferentiability, new necessary optimality conditions for unconstrained interval optimization problems with nondifferentiable interval-valued functions are derived. Moreover, the authors show a necessary condition for unconstrained weak sharp minima of an extended interval-valued functions in terms of the proposed notion of subdifferentiability.

In the work "The generalized conditional gradient method for composite multiobjective optimization problems on Riemannian manifolds" by X. Li, X. Ge, and K. Tu, a class of composite multiobjective optimization problems is considered. The feasible set defined on Riemannian manifolds is supposed to be closed and convex. In this paper, a generalized conditional gradient method with two step size strategies, including Armijo step size and the nonmonotone line search step size is developed. The global convergence result is established under some appropriate conditions. Furthermore, the iteration-complexity bound for composite multiobjective optimization problems is presented on Riemannian manifolds.

The special issue ends with the paper "Weak separation functions constructed by Gerstewitz and topical functions with applications in Conjugate Duality" by C. Yao and C. Tammer that aims to construct some nonlinear weak separation functions in image space analysis by virtue of the Gerstewitz and topical functions. Employing these separation functions, a framework of conjugate type duality for constrained vector optimization problems is developed. The primal problem is scalarized and then the separation functions are applied to give a scalar dual problem. Equivalent characterizations of the zero duality gap as well as the strong duality are established via subdifferential calculus, separation properties and saddle point assertions.

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