J. Nonlinear Var. Anal. 8 (2024), No. 3, pp. 369-370 Available online at http://jnva.biemdas.com https://doi.org/10.23952/jnva.8.2024.3.01

## EDITORIAL A SPECIAL ISSUE ON MATHEMATICAL OPTIMIZATION AND APPLICATIONS

The theory of mathematical optimization is concerned with the study of the minimization of mathematical functions. Usually, the optimization variables are subject to some side conditions or constraints. Optimization methods are now under spotlight of research due to their great utility in diverse areas, such as finance, engineering, and statistics. In particular, machine learning has been in recent years in the forefront of significant research activities. A key component of these research developments is the interplay with optimization methods.

This special issue focuses on recent research trends on the methods of mathematical optimization and their applications, aiming to push the developments of mathematical optimization for real-world problems. This special issue covers several hot topics. Details are presented below.

The paper "A scalable sphere-constrained magnitude-sparse SAR imaging" by M. Jiang, J. Qu, J. Ding, and J. Liang, establishes a sphere-constrained magnitude-sparsity SAR imaging model to enhance the SAR imaging quality with high efficiency. This paper also proposes a non-convex non-smooth optimization method, which can be accelerated by stochastic average gradient acceleration to be scalable with large-scale problems. Numerical experiments are conducted with point-target and extended-target simulations.

In the contribution, entitled "Quasi-subgradient methods with Bregman distance for quasi-convex feasibility problems" by Y. Hu, J. Li, Y. Liu, and C.K.W. Yu, a quasi-convex feasibility problem was studied. A unified framework of Bregman quasi-subgradient methods was presented for solving the problem. The convergence theory, including the global convergence, iteration complexity, and convergence rates, of the Bregman quasi-subgradient methods with several general control schemes was obtained.

L. Liu and S.Y. Cho, in "A Bregman projection algorithm with self adaptive step sizes for split variational inequality problems involving non-Lipschitz operators", discussed a split variational inequality problem governed by pseudomonotone and not necessarily Lipschitz continuous operators. They introduced a Bregman projection algorithm and presented the convergence analysis in the framework of Hilbert spaces. They also provided some numerical experiments to support their convergence theorems.

The paper "The level-set subdifferential error bound via Moreau envelopes" by Y. Wang, S. Li, M. Li, and X. Li is devoted to the behaviour of the level-set subdifferential error bound via Moreau envelopes under suitable assumptions. This paper provides an example that the Moreau

370 EDITORIAL

envelope does not have the Kurdyka-Łojasiewicz property but has the level-set subdifferential error bound when the original function does not satisfy the Kurdyka-Łojasiewicz property property but only the level-set subdifferential error bound.

In the paper "A fast and effective algorithm for sparse linear regression with  $\ell_p$ -norm data fidelity and elastic net regularization" by Y. Xiao, J. Shen, Y. Ding, M. Shi, and P. Li, the authors proposed a preconditioned proximal point algorithm to solve their model by adding a proximal term. In theory, they analyzed the consistency between the solution of the surrogate model and the original model. They also presented a large number of numerical experiments on high-dimensional simulated and real examples fully verify that their proposed algorithm is superior to ADMM in terms of calculation accuracy and speed.

In the paper "A parameterized three-operator splitting algorithm for non-convex minimization problems with applications" by L. Miao, Y.C. Tang, and C. Wang, the authors proposed a parameterized three-operator splitting algorithm to solve non-convex minimization problems with the sum of three non-convex functions, where two of them have Lipschitz continuous gradients. The authors established the convergence of the proposed algorithm under the Kurdyka-Łojasiewicz assumption by constructing a suitable energy function with a non-increasing property.

In the article "A multi-step inertial asynchronous sequential algorithm for common fixed point problems" by F. Su, L. Liu, X.H. Li, and Q.L. Dong, a multi-step inertial asynchronous sequential algorithm for common fixed point problems of nonexpansive operators was introduced and the weak convergence of the proposed algorithm was obtained. The application to linear systems was presented by combining the Kaczmarz method.

The special issue ends with the paper "Robust variational inequalities governed by curvilinear integral functionals" by S. Treanţă and J.C. Yao that mainly states that generalized convexity and the Fréchet differentiability assumption associated with curvilinear integral type functionals represent some mathematical tools for establishing various connections between the solutions.

We, as guest editors, would like to thank all the authors who made a contribution to this special issue and all the reviewers who kindly accepted the invitation to provide their expertise and gave constructive comments.

Jingwei Liang Shanghai Jiao Tong University, Shanghai, China E-mail address: jingwei.liang@sjtu.edu.cn

> Xiaolong Qin Zhejiang Normal University, Jinhua, China E-mail address: qxlxajh@163.com

Jen-Chih Yao China Medical University, Taichung, Taiwan E-mail address: yaojc@mail.cmu.edu.tw