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EDITORIAL
A SPECIAL ISSUE DEDICATED TO THE BLESSED MEMORY OF PROF. DR. HABIL. ALFRED GÖPFERT

FIGURE 1. Prof. Dr. habil. Alfred Göpfert in 2016.

This special issue is dedicated to commemorating and celebrating the remarkable accomplishments of Prof. Dr. Habil. Alfred Göpfert, whose legacy continues to inspire us despite his passing in January 2023. Alfred Göpfert embarked on his illustrious scientific journey at the

Institute of Mathematics, University of Leipzig, in 1960, where he laid the cornerstone of his expansive mathematical expertise. Under the tutelage of Herbert Beckert, he earned his doctoral degree in 1962, specializing in analysis and partial differential equations. Subsequently, in 1973, Alfred Göpfert achieved habilitation with his pioneering work on "Optimization problems with parabolic and elliptic differential equations" at the University of Leipzig. Concurrently, his fascination with convex analysis and optimization theory blossomed, propelling him to become a leading figure in academia and research within this domain for decades. In 1974, he was appointed Professor of Mathematics at the Technical University Leuna-Merseburg. During his tenure as the last rector of the Technical University Leuna-Merseburg from 1992 to 1993, Alfred Göpfert played a pivotal role in its administrative reformation. His transition to Halle marked a significant chapter in his career, where he contributed substantially to the restructuring of the Institute of Mathematics. Serving as a professor of Convex Analysis and Optimization at Martin Luther University Halle-Wittenberg from 1992 to 1999, he enriched the academic landscape before assuming emeritus status in 1999.

Alfred Göpfert's impact extended beyond the confines of academia; he ignited a passion for mathematics in successive generations through his erudition, innovative research, and affable demeanor. Renowned for his openness to dialogue, he imparted invaluable insights to his students and colleagues, fostering an environment conducive to intellectual exploration. His profound expertise and unwavering composure underpinned his scholarly endeavors. His seminal contributions reverberated across international platforms, showcased through his prolific presentations at conferences and numerous publications in esteemed journals and books. Continuously pushing the boundaries of research, particularly in the realms of optimization in generalized spaces and its underpinnings from functional analysis, he inspired novel avenues of inquiry. Alfred Göpfert's legacy endures as a testament to his excellence as an academic mentor and researcher. Colleagues fortunate enough to collaborate with him benefitted from his adeptness in elucidating intricate interconnections across diverse mathematical disciplines. Conversations with Alfred Göpfert invariably ignited a thirst for further mathematical exploration, leaving an indelible mark on all who had the privilege of engaging with him.

In commemorating the life and achievements of Alfred Göpfert, this special issue serves as a testament to his enduring legacy, inspiring generations of mathematicians to pursue excellence and innovation in their scholarly pursuits.

This special issue is comprised of twelve articles whose contributions are as follows:

The article titled "Noncoercive elliptic bilateral variational inequalities in the homogeneous Sobolev space $D^{1,p}(\mathbb{R}^n)$" by S. Carl is dedicated to a comprehensive study of noncoercive variational inequalities in which the underlying space lacks the necessary required compact embedding. The primary contribution of the manuscript lies in the development of a novel existence theory for the aforementioned variational inequality and in establishing the compactness of the solution set.

The aim of X. Zhao, H. Zhang, and Y. Yao in their contribution titled "An inexact nonmonotone projected gradient method for constrained multiobjective optimization" is to explore an inexact projected gradient approach integrated with a non-monotone line search strategy tailored for smooth multiobjective optimization problems. The authors introduce a novel nonmonotone line search technique, demonstrating that each accumulation point of the sequence generated by this method is Pareto stationary, and conduct an in-depth analysis of the convergence rate of

the algorithm. They further establish that, for a convex objective function, the convergence of the sequence to a weak Pareto optimal point is guaranteed.

The primary focus of M. Durea and E. Florea in their contribution titled "Subdifferential calculus and ideal solutions for set optimization problems" is to develop fundamental calculus rules for certain subdifferential constructions related to set-valued maps between normed vector spaces. They utilize these results to derive optimality conditions for a specific class of solutions to set optimization problems.

D. Dörfler and A. Löhne, in their article titled "A polyhedral approximation algorithm for recession cones of spectrahedral shadows," address the approximate computation of recession cones associated with spectrahedral shadows by using polyhedral cones. The main contribution of this article lies in presenting two iterative algorithms for computing outer and inner approximations with a user-defined level of accuracy. The authors demonstrate the feasibility and finiteness of both algorithms, supported by numerical examples.

The article titled "Closed convex sets that are both Motzkin and generalized Minkowski" by J. E. Martínez Legaz and C. Pintea explores sets that exhibit both Motzkin decomposability and a generalization of Minkowski properties, briefly referred to as MdgM sets. The authors demonstrate the existence of well-defined fixed points for multivalued functions defined on MdgM sets. The first set of existence results draws upon Kakutani's fixed point theorem, providing a robust foundation for understanding the dynamics of multivalued functions on MdgM sets. On the other hand, the second class of results builds upon both Brouwer's fixed point theorem and the Banach contraction principle, further enriching the analysis with additional insights into single-valued self-functions of MdgM sets.

J. Cen, S. Migorski, C. Vetro, and S. Zeng in their article titled "Stability analysis for a contaminant convection-reaction-diffusion model of recovered fracturing fluid," conduct a thorough study of the stability analysis for a contaminant convection-reaction-diffusion model of the recovered fracturing fluid (RFFM, for short), which couples a nonlinear and nonsmooth stationary incompressible Navier-Stokes equation with a multivalued friction boundary condition, and a nonlinear reaction-diffusion equation with mixed Neumann boundary conditions.

In the article "Nonlinear strict cone separation theorems in real normed spaces," authors C. Günther, B. Khazayel, and C. Tammer focus on deriving novel results regarding the separation of two cones, which may not necessarily be convex, by a convex cone or conical surface in real reflexive normed spaces. The authors establish connections between their new cone separation theorem and existing results in the literature by introducing a new characterization of the algebraic interior of augmented dual cones in real normed spaces.

The article "Convergence of a New nonmonotone memory gradient method for unconstrained multiobjective optimization via robust approach" authored by Y. Bai, J. Chen, L. Tang, and T. Zhang, introduces a new non-monotone gradient algorithm for unconstrained multiobjective optimization challenges. Employing a blend of conjugate technique and robust approach, the method stands out for its memory gradient property. The article establishes the convergence of the iterative sequence generated by this method towards a Pareto critical point of the multiobjective optimization problem. Additionally, numerical experiments are presented to demonstrate the effectiveness of the proposed methodology.

In the article titled "Normalized duality mappings and projections in some Bochner spaces", L. Cheng, A. A. Khan, J. Li, and C. Tammer employ the analytic representation of the normalized duality mapping to establish and substantiate certain non-convex properties linked to this mapping in uniformly convex and uniformly smooth Bochner spaces. Variational characterizations of the projection map in Bochner spaces are also given.

In their contribution titled "Characterization of $\mathscr{E}$-Benson proper efficient solutions of vector optimization problems with variable ordering structures in linear spaces," J. W. Peng, W. B. Wei, D. Ghosh, and J. C. Yao focus on introducing novel $\mathscr{E}$-Benson proper efficient elements for subsets within a linear space, considering variable ordering maps. Their work entails a comprehensive exploration of $\mathscr{E}$-Benson proper efficient solutions of vector optimization problems under variable ordering structures.

The focus of D. Dörfler and A. Löhne in their contribution titled "Convex sets approximable as the sum of a compact set and a cone" is to introduce a significant class of convex sets, termed approximately Motzkin-decomposable. These sets can be approximated as the Minkowski sum of a compact convex set and a closed convex cone, measured in the Hausdorff distance. The authors provide a characterization of these sets based on their support functions. They demonstrate that a set is approximately Motzkin-decomposable if and only if its support function exhibits continuity over a closed domain.

In conclusion, we extend our heartfelt gratitude to all the authors whose contributions enriched this special issue, as well as to the reviewers whose thorough, timely, and insightful feedback significantly enhanced the quality of the articles.

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